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Analytical and experimental response time to flow rate step along a counter flow double pipe heat exchanger

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Abstract

Temperatures transient response along a tubular counter flow heat exchanger is investigated when mass flow rate is subjected to sudden change. The dynamic behaviour is approximated by a first order response with a time constant. The hot fluid subjected to the flow rate step presents a decreasing linear time constant according to flow axis. The cold fluid, which is not submitted to the flow rate step, shows two types of transient response. The first one corresponds to a linear spatial increasing function of time constant while the second one presents a uniform time constant along the heat exchanger. Considering the conditions of the transient response on the boundaries of the heat exchanger, analytical expressions of time constants are obtained for each case. The comparison of the theoretical results with experimental data allows to validate the analytical expressions, which depend on initial and final steady-states. The influence of the flow rate step magnitude on transient behaviour is studied by maintaining the initial steady-state. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Heat exchangers are extensively used in all kinds of industries to control heat transfer and temperature of chemical mixtures in reactors. They are commonly connected to other thermal equipments and this can affect the parameters such as inlet temperatures and mass flow rates. It is necessary to explore the unsteady-state of the thermal processes when real time control, state computation, optimisation and rational use of energy are investigated. The transient behaviour can be investigated in two ways. The first one corresponding to numerical techniques is time-consuming in term of computation. The second one corresponds to analytical methods, which present the advantage to assess immediately the transient response when design of heat exchanger is needed. In addition, the advanced control of industrial systems based on internal models becomes efficient and robust when a simple model is used. In the case of heat exchangers, the temporal response can be described as a time delay added to a rising time [1–3]. Pierson and Padet [1,2] present analytical expression of this approach. A generalisation of this method was recently given in references [4–8]. This method assumes that the two fluids are characterised by the same time constant and the lag time, which have the same values for both fluids and are supposed to be spatially independent. A general formulation and modelling of the transient behaviour of heat exchangers is also discussed in [9–11].

In this paper, the spatial variation of transient response of temperatures along a counter current heat exchanger is presented when a flow rate step is applied to the internal hot fluid. Our interest focuses on unsteady-state along the heat exchanger without limiting the observations to only the inlet and outlet of the thermal device. Analytical expressions of the time constant for each fluid are derived when the heat exchanger is subjected to a step of mass flow rate. The influence of initial and final flow rates is investigated. The transient response to positive and negative flow rate step is also presented.

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Nomenclature

a	wall thickness (m)
A	heat transfer area (m^2)
Bi	Biot number
C	heat capacity ($J\ K^{-1}$)
C_p	specific heat ($J\ K^{-1}\ kg^{-1}$)
D	diameter (m)
h	heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
k	thermal conductivity ($W\ m^{-1}\ K^{-1}$)
L	heat exchanger length (m)
$\dot{m}$	mass flow rate ($kg\ s^{-1}$)
N	dimensionless number
NTU	number of transfer units
Nu	Nusselt number
Pr	Prandtl number
R	radius (m)
Re	Reynolds number
t	time (s)
T	temperature (K)
V	mean velocity ($m\ s^{-1}$)
Vol	volume (m^3)
x	axial position (m)

Greek symbols

δ	boundary layer thickness (m)
η	kinematic viscosity ($m^2\ s^{-1}$)
μ	dynamic viscosity ($kg\ m^{-1}\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
τ	time constant (s)

Subscripts

c	cold fluid
e	outer tube
h	hot fluid
i	inner tube
n	input stream
out	output stream
w	separating wall

Superscripts

0	initial conditions
∞	final conditions
$*$	dimensionless form

2. Description and modelling

2.1. Experimental device and description

In order to characterise the spatial distribution of unsteady-state temperatures field in both fluids, the instrumentation of the experimental device reveals to be quite sufficient. Fig. 1 shows the experimental set up used to validate the theoretical results. Temperature probes are alternatively placed along the tubular insulated heat exchanger and are distant of 0.36 m for both fluids. Their diameter is equal to 0.001 m and their rising time is less than 0.5 s. The sensors are linked to a data acquisition card inserted into a computer, which enables the acquirement and the temperatures display along the heat exchanger. The inner tube is in copper and the

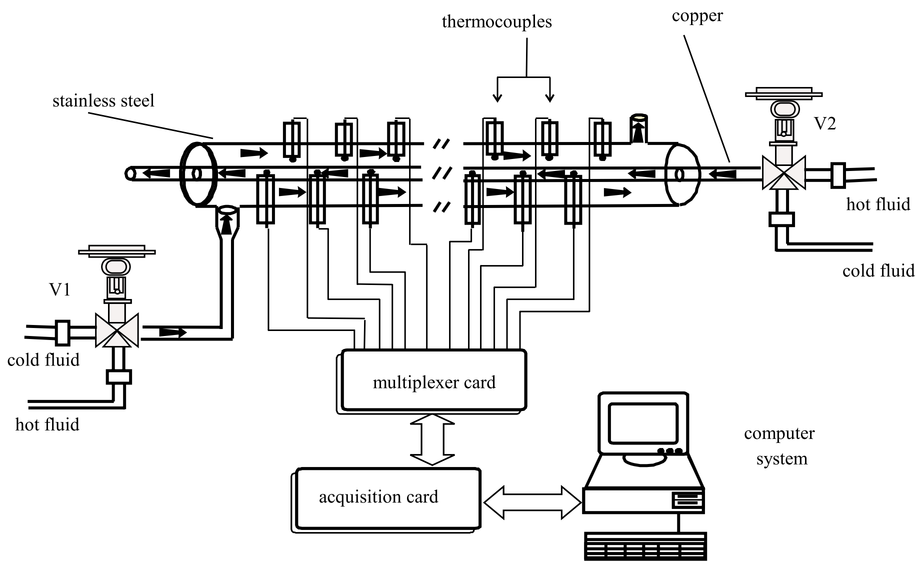


Fig. 1. Experimental set up with its electronic cards and system acquisition.

Table 1
Physical and geometrical characteristics of inner and outer tubes

	k (W K^{-1} m^{-1})	C_p (J kg^{-1} K^{-1})	ρ (kg m^{-3})	D (m)	a (m)	L (m)
Inner tube	384	394	8900	0.02	0.001	4.5
Outer tube	45	490	7850	0.04	0.003	4.5

outer one is in steel. The physical and geometrical characteristics of the tubes are reported on Table 1. The two streams are water–water flow.

2.2. Assumptions and modelling

This paper deals with the transient response of a tubular counter current heat exchanger when a sudden flow rate step applied one of the streams. The mathematical model used in this study is proposed by Patankar et al. [12]. The assumptions made in our case are:

- fluids are in turbulent flow,
- heat conduction along the flow axis is neglected,
- fluids are incompressible and single phased,
- thermophysical properties of the fluids are assumed to be constant,
- the separating wall is assumed to be isothermal along the radial axis.

The Biot numbers of both sides, $Bi_{h,c} = h_{h,c}a/k_w$, corresponding to experimental conditions are lower than 0.01 ($h_{h,c} < 5000$ W m^{-2} K^{-1}).

In order to derive the governing differential equations, the heat exchanger is subdivided in several elemental volumes of length dx. Energy balance applied to a differential volume of the hot fluid leads to the first equation of dimensionless system (1) after simplification and rearrangement:

$$\begin{aligned} \frac{\partial T_h^*}{\partial t^*} &= V^* \frac{\partial T_h^*}{\partial x^*} + V^* N_h (T_w^* - T_h^*), \\ \frac{\partial T_c^*}{\partial t^*} &= -\frac{\partial T_c^*}{\partial x^*} + N_c (T_w^* - T_c^*), \\ \frac{\partial T_w^*}{\partial t^*} &= V^* C_h^* N_h (T_h^* - T_w^*) + C_c^* N_c (T_c^* - T_w^*). \end{aligned} \quad (1)$$

The boundary conditions are:

$$\begin{aligned} T_h^*(1, t^*) &= T_{h,in}^* = 1, \\ T_c^*(0, t^*) &= T_{c,in}^* = 0 \end{aligned} \quad (2)$$

and the step is applied on V^*

$$V^*(t^*) = \begin{cases} V^{*0} & \text{for } t^* < t_0^* = 0, \\ V^{*\infty} & \text{for } t^* \geqslant t_0^* = 0 \end{cases}. \quad (3)$$

The dimensionless temperatures are taken in the following form

$$T_{h,c,w}^*(x^*, t^*) = \frac{T_{h,c,w}(x^*, t^*) - T_{c,in}}{T_{h,in} - T_{c,in}}. \quad (4)$$

The different parameters are defined as:

$$x^* = \frac{x}{L}, \quad t^* = t\frac{V_c}{L}, \quad V^* = \frac{V_h}{V_c}, \quad C_h^* = \frac{C_h}{C_w}, \quad C_c^* = \frac{C_c}{C_w},$$
$$N_h = \frac{h_h A_h}{\dot{m}_h C_{ph}} \quad \text{and} \quad N_c = \frac{h_c A_c}{\dot{m}_c C_{pc}}.$$

The heat capacities are defined as $C_{h,c,w} = \rho_{h,c,w} \mathrm{Vol}_{h,c,w} C_{ph,c,w}$.

Note that when the step is applied on the mass flow rate, the characteristic time ($\tau_\delta = \delta^2/\eta$) of the new boundary layer establishment is very smaller than the temperatures response of the heat exchanger ($\tau_\delta \ll 1$ s). Therefore, the step can also be considered on the mean velocity of the fluid.

In this dimensionless formulation, the groups N_h and N_c are very similar to the number of transfer units used in steady-state NTU method [13,14]. The NTU is linked to the global heat transfer while N_h and N_c depend on the convective heat transfer coefficients. They can also be written as a function of Stanton number and geometrical characteristics of the heat exchanger. The modified Colburn correlation is widely used in the literature to assess the heat transfer coefficient for circular ducts and was given by Sieder and Tate [15].

$$Nu_h = \frac{h_h D_i}{k_h} = 0.027 Re_h^{4/5} Pr_h^{1/3} \left(\frac{\mu}{\mu_w}\right)^{0.14}, \quad (5)$$

where μ/μ_w is the ratio of dynamic viscosity at the flow centre and at the wall neighbourhood. For annulus duct, other kind of correlations are used. Kawamura [16] proposed two correlations for turbulent annular flow as a function of radius ratio ($r_0^* = r_e/r_i$)

$$Nu_c = 0.022 \varphi_i Re_c^{0.8} Pr_c^{0.5}. \quad (6)$$

The accommodation factor φ_i represents the ratio of the shear stress on the inner wall to the average shear stress on the inner and outer walls. More details of these correlations are given in [16].

2.3. Transient behaviour

The transient response to flow rate step is instantaneous and can be approximated by an exponential response like a first order system as described in reference [17]. The temperatures of the two streams and the separating wall along the heat exchanger could be written as

$$T_{h,c,w}^*(x^*, t^*) = T_{h,c,w}^{*0}(x^*) + \left(T_{h,c,w}^{*\infty}(x^*) - T_{h,c,w}^{*0}(x^*)\right)$$

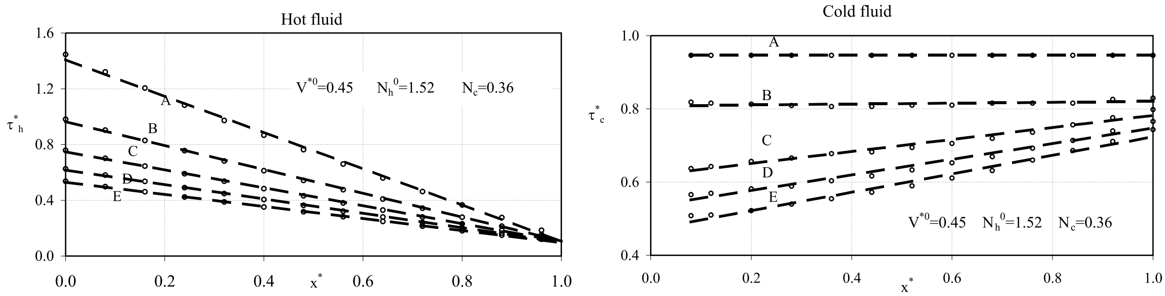


Fig. 2. Time constant of both fluids along the heat exchanger for different values of step magnitude: (A) $V^{*\infty} = 0.67$, $N_{\rm h}^{\infty} = 1.40$; (B) $V^{*\infty} = 0.89$, $N_{\rm h}^{\infty} = 1.32$; (C) $V^{*\infty} = 1.12$, $N_{\rm h}^{\infty} = 1.27$; (D) $V^{*\infty} = 1.34$, $N_{\rm h}^{\infty} = 1.22$; (E) $V^{*\infty} = 1.56$, $N_{\rm h}^{\infty} = 1.18$.

$$\times\left\{1-\exp\left(-\frac{t^*}{\tau^*_{\rm h,c,w}(x^*)}\right)\right\}. \qquad (7)$$

In these expressions, the variations of the time constants $\tau^*_{\rm h}$, $\tau^*_{\rm c}$ and $\tau^*_{\rm w}$ appear as a functions of space. The time constants along the heat exchanger extracted from the numerical resolution of system (1), obtained by the McCormack method [18], for different flow rate step magnitudes is reported in Fig. 2. The first graph of this figure is relative to the hot fluid and the second one corresponds to the cold fluid. Time constant of fluid not subjected to the flow rate step shows two types of transient behaviour:

1. time constant is an increasing straight line and in this case we assume that $\tau^*_{\rm w}(0) \cong \tau^*_{\rm h}(0) \cong \tau^*_{\rm c}(0)$,
2. time constant is assumed to be uniform along the heat exchanger and then can be expressed as $\frac{{\rm d}\tau^*_{\rm c}}{{\rm d}x^*}(x^*) \approx 0.$

3. Analytical approach

3.1. First case

3.1.1. Hot fluid

Substituting expressions of spatial and temporal derivatives of $T^*_{\rm h}(x^*, t^*)$, obtained from relation (7), into the first equation of system (1), and eliminating relations at steady-state of $T^{*0}_{\rm h}(x^*)$ and $T^{*\infty}_{\rm h}(x^*)$ leads to the following expression at time $t^* = \tau^*_{\rm h}(x^*)$

$$\begin{aligned}\frac{T^{*\infty}_{\rm h}(x^*)-T^{*0}_{\rm h}(x^*)}{{\rm e}\tau^*_{\rm h}(x^*)} = &-\frac{V^{*\infty}}{\rm e}\left\{\left(\frac{{\rm d}T^{*\infty}_{\rm h}}{{\rm d}x^*}(x^*)-\frac{{\rm d}T^{*0}_{\rm h}}{{\rm d}x^*}(x^*)\right)\right.\\ &\left.+\left(\frac{T^{*\infty}_{\rm h}(x^*)-T^{*0}_{\rm h}(x^*)}{\tau_{\rm h}(x^*)}\right)\frac{{\rm d}\tau^*_{\rm h}(x^*)}{{\rm d}x}\right\}\\ &-V^{*\infty}N^{\infty}_{\rm h}\left\{\left(T^{*\infty}_{\rm w}(x^*)-T^{*0}_{\rm w}(x^*)\right)\right.\\ &\times\exp\left(-\frac{\tau^*_{\rm h}(x^*)}{\tau^*_{\rm w}(x^*)}\right)-\frac{1}{\rm e}\left(T^{*\infty}_{\rm h}(x^*)\right.\\ &\left.\left.-T^{*0}_{\rm h}(x^*)\right)\right\}. \qquad (8)\end{aligned}$$

In expression (8), the final value of $N^{\infty}_{\rm h}$ takes into account the heat capacity rate variation of the hot fluid and the convective heat transfer coefficient change. e designates the exponential number. As indicated previously, time constants are spatially linear. Therefore, the determination of $\tau^*_{\rm h}$ at two points is sufficient to characterise the transient behaviour along the heat exchanger. Particularly, time constants will be evaluated at the inlet and the outlet of the heat exchanger ($x^* = 0$ and $x^* = 1$). It is worth noting that temperature of the cold fluid is maintained constant at the inlet ($x^* = 0$). The temperature evolution of the separating wall at this position is similar to the dynamic of the cold fluid. This result is obtained from the second equation of system (1). This statement is important in the determination of $\tau^*_{\rm h}$ from differential Eq. (8) at $x^* = 0$.

Using Eq. (8) at the neighbourhood of $x^* = 0$, the following differential equation is obtained:

$$\begin{aligned}&\frac{{\rm d}\tau^*_{\rm h}}{{\rm d}x^*}(x^*)-\left\{\frac{(1-(N^{\infty}_{\rm h}/N^0_{\rm h}))T^{*0}_{\rm h,out}}{((1/N^0_{\rm h})+(1/N_{\rm c})V^{*0}C^*)(T^{*\infty}_{\rm h,out}-T^{*0}_{\rm h,out})}\right\}\tau^*_{\rm h}(x^*)\\ &\quad=-\frac{1}{V^{*\infty}}, \qquad (9)\end{aligned}$$

where $C^* = C^*_{\rm h}/C^*_{\rm c}$.

Note that the differential equation of (9) $\tau^*_{\rm h}$ is still valid around $x^* \sim 0$ and can be written as

$$\frac{{\rm d}\tau^*_{\rm h}}{{\rm d}x^*}(0)-\lambda\tau^*_{\rm h}(0) = -\frac{1}{V^{*\infty}} \qquad (10)$$

with

$$\lambda = \left(\frac{1-(N^{\infty}_{\rm h}/N^0_{\rm h})}{(1/N^0_{\rm h})+(1/N_{\rm c})V^{*0}C^*}\right)\left(\frac{T^{*0}_{\rm h,out}}{T^{*\infty}_{\rm h,out}-T^{*0}_{\rm h,out}}\right). \qquad (11)$$

Writing $({\rm d}\tau^*_{\rm h}/{\rm d}x^*)(x^*) = \tau^*_{\rm h,in}-\tau^*_{\rm h,out}$, the linear expression of response time of hot fluid is finally obtained as

$$\tau^*_{\rm h}(x^*) = \frac{-\left(1-\lambda V^{*\infty}\tau^*_{\rm h,in}\right)x^*+\left(1+V^{*\infty}\tau^*_{\rm h,in}\right)}{(1+\lambda)V^{*\infty}}. \qquad (12)$$

In expression (12), $\tau^*_{\rm h,in}$ corresponding to time constant at the inlet is obtained from the differential equation of the separating wall temperatureat $x^* = 1$

$$\tau^*_{\rm h,in} = (T^{*\infty}_{\rm w}(1) - T^{*0}_{\rm w}(1))/(C^*_{\rm c}(V^{*\infty}N^{\infty}_{\rm h}C^* + N_{\rm c}) \times (T^{*\infty}_{\rm w}(1) - T^{*0}_{\rm w}(1)) + eN_{\rm c}(T^{*0}_{\rm c,out} - T^{*\infty}_{\rm c,out})). \quad (13)$$

Initial and final wall steady-state temperature $T^{*0}_{\rm w}(1)$ and $T^{*\infty}_{\rm w}(1)$ are deduced from the following expressions

$$T^{*0}_{\rm w}(1) = \frac{V^{*0}N^{0}_{\rm h}C^* + N_{\rm c}T^{*0}_{\rm c,out}}{V^{*0}N^{0}_{\rm h}C^* + N_{\rm c}},$$
$$T^{*\infty}_{\rm w}(1) = \frac{V^{*\infty}N^{\infty}_{\rm h}C^* + N_{\rm c}T^{*\infty}_{\rm c,out}}{V^{*\infty}N^{\infty}_{\rm h}C^* + N_{\rm c}}. \quad (14)$$

It is worth noting that in some cases, $\tau^*_{\rm h,in}$ can be affected when the dynamic of input stream temperature is due to the heat transfer before going through the heat exchanger. Therefore expression (13) is valid for insulated outside tubes of the heat exchanger.

3.1.2. Cold fluid

The same approach was applied to determine the straight line of the response time $\tau^*_{\rm c}(x^*)$ as a function of space. Introducing in the second equation of system (1) temporal and spatial derivatives of the cold fluid temperature in the same way as for the hot fluid, and taking time $t^* = \tau^*_{\rm c}(x^*)$ and $x^* = 0$, we obtain

$$0 = -\left\{\left(\frac{{\rm d}T^{*\infty}_{\rm c}}{{\rm d}x^*}(0) - \frac{{\rm d}T^{*0}_{\rm c}}{{\rm d}x^*}(0)\right)\left(1 - \frac{1}{e}\right)\right\} + N_{\rm c}\left\{\left(T^{*\infty}_{\rm w}(0) - T^{*0}_{\rm w}(0)\right)\left(1 - \exp\left(-\frac{\tau^*_{\rm c}(0)}{\tau^*_{\rm w}(0)}\right)\right)\right\}. \quad (15)$$

Expression (15) leads to $\tau^*_{\rm c}(0) = \tau^*_{\rm w}(0)$.

The cold fluid temperature is maintained constant at the inlet of the heat exchanger. At this position the dynamic of the separating wall is linked only to the hot fluid temperature. Recalling $\tau^*_{\rm h}(0)$ from (12), the response time of the cold fluid at its inlet ($x^* = 0$) can be written as follows:

$$\tau^*_{\rm c}(0) = \frac{1 + V^{*\infty}\tau^*_{\rm h,in}}{(1+\lambda)V^{*\infty}}. \quad (16)$$

Noting that the temperature of the hot fluid is maintained constant at the neighbourhood of $x^* = 1$ and that $\tau^*_{\rm w}$ is insignificant compared to $\tau^*_{\rm c}$, we obtain at time $t = \tau^*_{\rm c}(x^*)$

$$\frac{{\rm d}\tau^*_{\rm c}}{{\rm d}x^*}(x^*) + N_{\rm c}\left(\frac{T^{*\infty}_{\rm w}(1) - T^{*0}_{\rm w}(1)}{T^{*\infty}_{\rm c,out} - T^{*0}_{\rm c,out}}\right)\tau^*_{\rm c}(x^*) = 1, \quad (17)$$

where $T^{*0}_{\rm w}(1)$ and $T^{*\infty}_{\rm w}(1)$ are defined by expressions (11). $\tau^*_{\rm c}(1) = \tau^*_{\rm c,out}$ is extracted from (17) and the characteristics of the straight line of the cold fluid time constant is finally obtained

$$\tau^*_{\rm c}(x) = \left(\frac{1}{1+\gamma}\right)\left\{1 - \gamma\tau^*_{\rm h,in} - \frac{\gamma}{(1+\lambda)V^{*\infty}}\right\}x^* + \left(\frac{1}{1+\lambda}\right)\left(\frac{1}{V^{*\infty}} + \tau^*_{\rm h,in}\right), \quad (18)$$

where

$$\gamma = N_{\rm c}\left(\frac{T^{*\infty}_{\rm w}(1) - T^{*0}_{\rm w}(1)}{T^{*\infty}_{\rm c,out} - T^{*0}_{\rm c,out}}\right). \quad (19)$$

Relations (11) and (19) show that λ and γ depend only on initial and final steady-state profiles of temperatures, which implies that the spatial variation of temporal behaviour of the heat exchanger can be obtained from steady-state temperatures only.

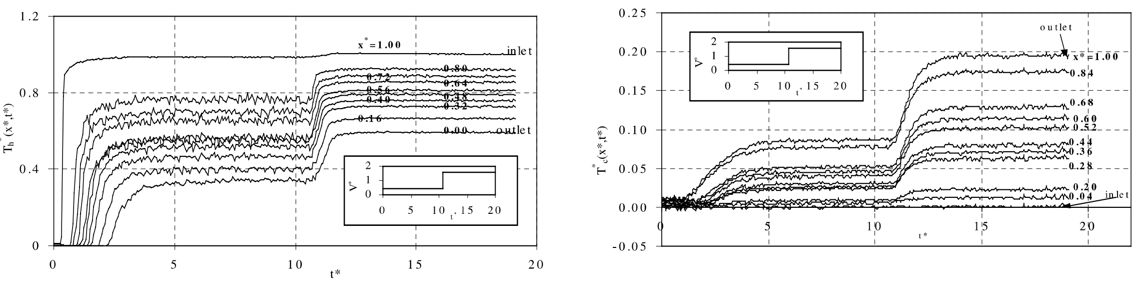
3.2. Second case

In this section, $\tau^*_{\rm c}$ is determined before expressing $\tau^*_{\rm h}$.

3.2.1. Cold fluid

By putting the spatial derivative of $\tau^*_{\rm c}$ equal to zero (${\rm d}\tau^*_{\rm c}/{\rm d}x^*(x^*) \approx 0$) and taking x^* at the neighbourhood of 1, we obtain

$$\frac{T^{*\infty}_{\rm c,out} - T^{*0}_{\rm c,out}}{\tau^*_{\rm c}(1)} = \left(\frac{{\rm d}T^{*\infty}_{\rm c}}{{\rm d}x^*}(1) - \frac{{\rm d}T^{*0}_{\rm c}}{{\rm d}x^*}(1)\right) + N_{\rm c}\left(T^{*\infty}_{\rm c,out} - T^{*0}_{\rm c,out}\right), \quad (20)$$



(a) (b)

Fig. 3. Temporal evolution of experimental dimensionless temperatures for different axial position.

$\tau_c^*(1)$ is then derived from (20) and corresponds to $\tau_c^*(x^*)$ since the dynamic of the cold fluid temperature is independent of the position in the heat exchanger

$$\tau_c^*(x^*) = \frac{1}{\gamma} = \frac{1}{N_c} \frac{T^{*\infty}_{c,out} - T^{*0}_{c,out}}{\left(T^{*\infty}_w(1) - T^{*0}_w(1)\right)}, \tag{21}$$

where γ is defined by the same expression (19) in the first case.

3.2.2. Hot fluid

At the neighbourhood of $x^* = 0$, equation of the separating wall for $t^* = \tau_h^*(0)$ allows to deduce the time constant of the hot fluid temperature at $x^* \sim 0$

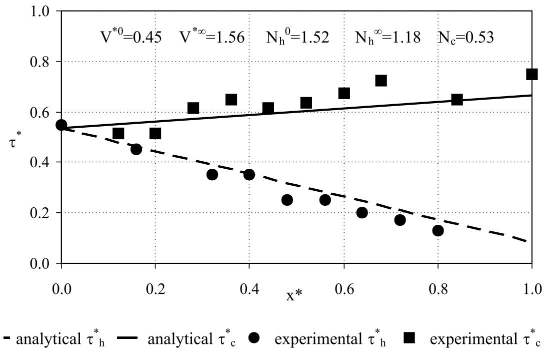


Fig. 4. Experimental and theoretical time constants of both fluids along the heat exchanger.

$$\tau_h^*(0) = -\frac{1}{\gamma} \ln \left\{ -\frac{V^{*\infty} N_h^{\infty} C^*}{e\left(\frac{\gamma}{C_c^*} - V^{*\infty} N_h^{\infty} C^* - N_c\right)} \times \left(\frac{T^{*\infty}_{h,out} - T^{*0}_{h,out}}{T^{\infty}_w(0) - T^0_w(0)} \right) \right\}. \tag{22}$$

Note that the response time of temperature of hot fluid at $x^* = 1$ given for the first case by relation (13) is also available for the second case. $\tau_h^*(x^*)$ is then obtained by

$$\tau_h^*(x^*) = \left(\tau_h^*(1) - \tau_h^*(0)\right)x^* + \tau_h^*(0), \tag{23}$$

where $\tau_h^*(1)$ and $\tau_h^*(0)$ are given, respectively, by expressions (13) and (22).

4. Calculations and discussion

Graphs (a) and (b) of Fig. 3 show, respectively, the temporal evolution of experimental dimensionless temperatures of hot and cold fluid. These graphs represent the start up of the heat exchanger ($t^* < t_0^*$) before applying a mass flow rate step ($t^* \geqslant t_0^*$). t_0^* corresponds to the time when the flow rate step is applied. Fig. 4 shows the good agreement between theoretical and experimental time constants along the heat exchanger for both fluids. The experimental results are extracted using the same technique of [1] and the theoretical results are obtained from the above mentioned analytical expres-

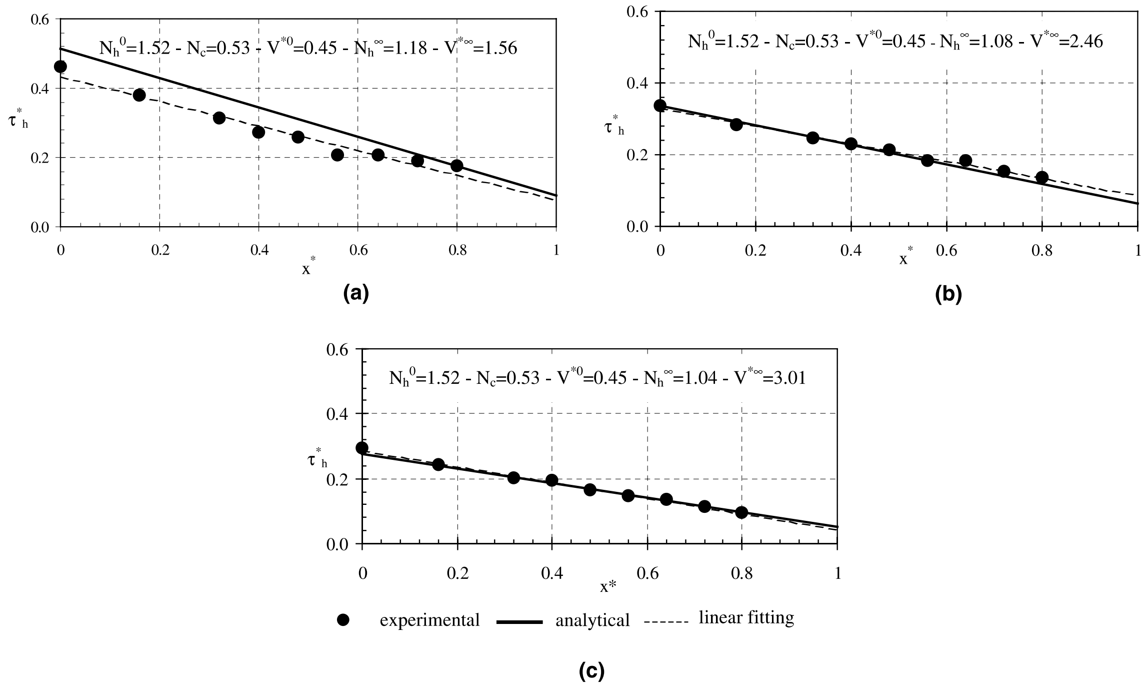


Fig. 5. Influence of flow rate step magnitude on the spatial profile of hot fluid time constant.

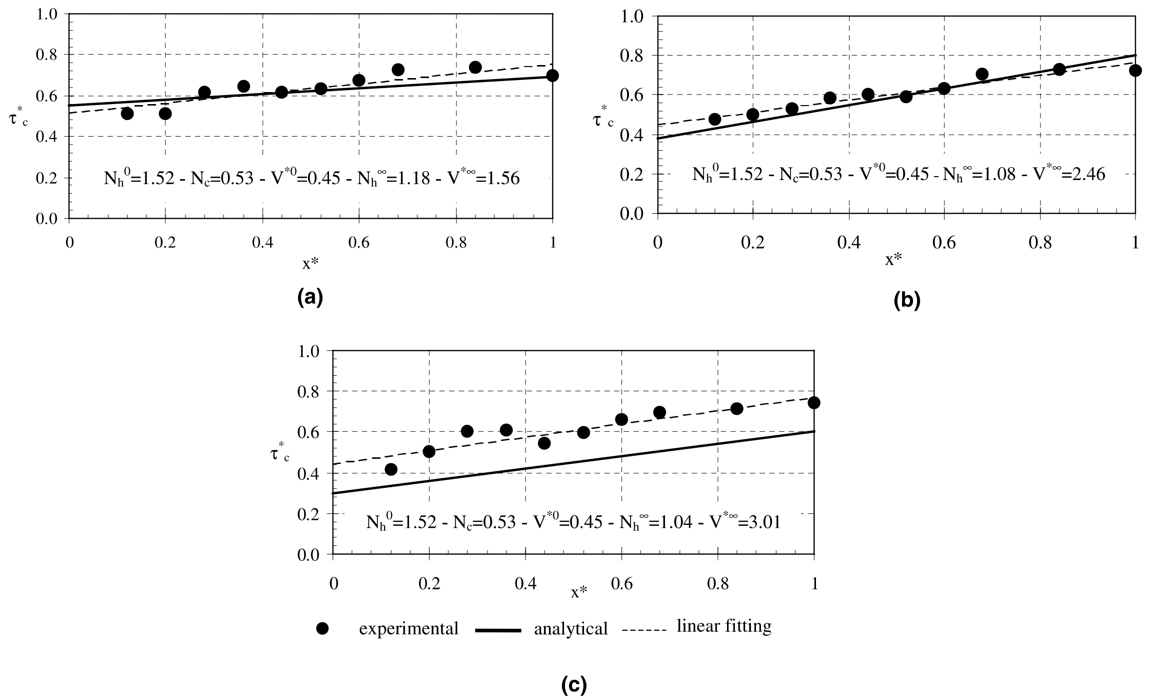


Fig. 6. Experimental and theoretical time constants of cold fluid along the heat exchanger for different values of flow rate step.

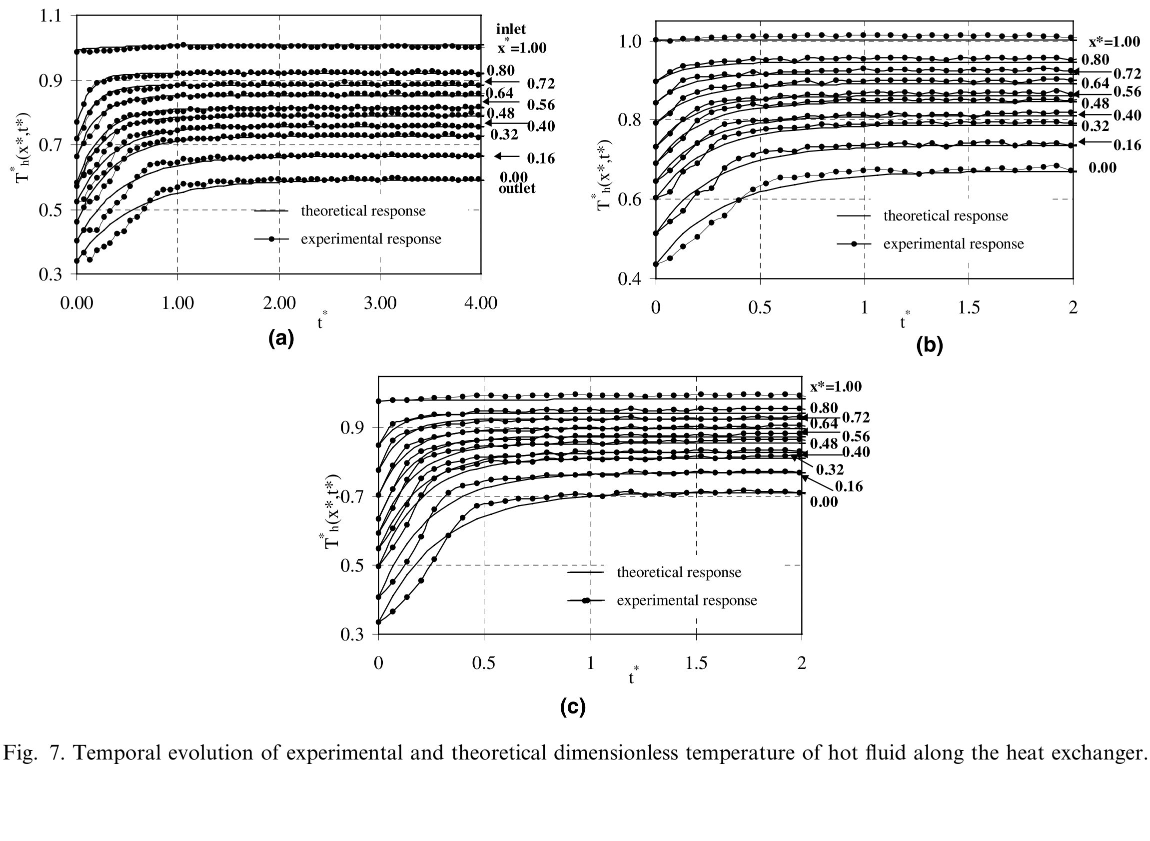


Fig. 7. Temporal evolution of experimental and theoretical dimensionless temperature of hot fluid along the heat exchanger.

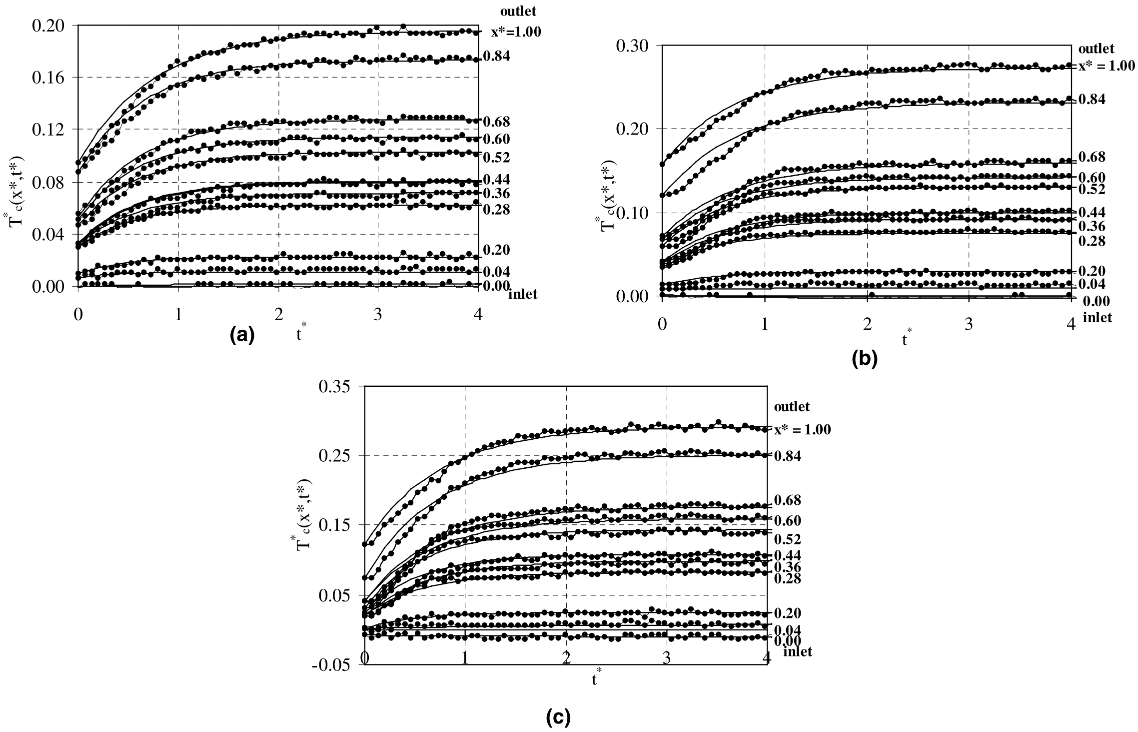


Fig. 8. Spatial and temporal evolution of experimental and theoretical dimensionless temperature of cold fluid for different flow rate step magnitude.

sions considering the time constant $\tau^*_{h,in}$ due to transient heat exchange between inner tube and environment before crossing the heat exchanger. The convective heat transfer coefficients are deduced from correlations presented previously. τ^*_h decreases linearly according to the flow direction while τ^*_c is an increasing function. Note that these results validate the expressions (12) and (18) of τ^*_h and τ^*_c corresponding to the first case where the time constant of cold fluid is not uniform along the heat exchanger. It is worth noting that in this case the variation of time constants of both fluids along the heat exchanger could reach 50%. In the same experimental conditions, the second case appears when the flow rate step takes weak positive values. This one can not be validated experimentally because of the small variation of the temperatures along the heat exchanger. These results show that experimental data corroborates the theoretical expressions of spatial variation of transient behaviour for both hot and cold fluid while a flow rate step is applied to the heat exchanger. Fig. 5 shows the experimental and theoretical time constants of the hot fluid temperature as a function of dimensionless axial position for different values of flow rate step magnitude with the same value of initial hot fluid flow rate. The time constant increases while step value of flow rate decreases. Theoretical and experimental results of hot fluid are in good agreement for the second and third graphs (B and C) of this figure. The graph A presents some difference about 15%. This is due probably to the uncertainty of the time constant of the hot fluid at the inlet (heat transfer before crossing the heat exchanger). In the same experimental conditions, Fig. 6 is obtained for cold fluid. Good agreement between experimental and theoretical results of cold fluid is shown in graphs (a) and (b), but the third one presents a difference about 20%. However, the slope of τ^*_c (x^*) increases with the flow rate step magnitude, which is in agreement with the numerical results presented previously. Fig. 7 shows

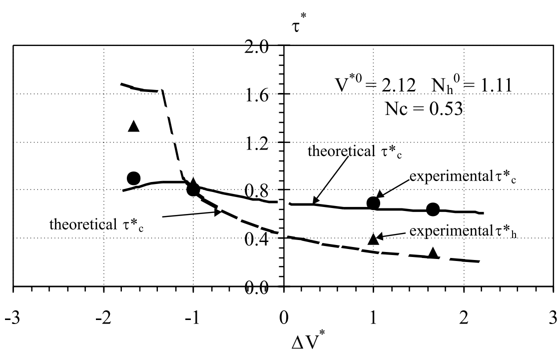


Fig. 9. Experimental and theoretical time constants of outlet fluids temperatures as a function of step magnitude.

the temporal evolution of theoretical and experimental temperatures of the hot fluid, for many values of step magnitude in the same experimental conditions as of Fig. 5. The theoretical and experimental temporal response of the cold fluid to flow rate step for different values of ΔV^* ($\Delta V^* = V^{*\infty} - V^{*0}$) is also reported on Fig. 8.

The experimental temperatures of Fig. 7 differ slightly from the theoretical results particularly at the outlet. The theoretical response is more accurate on Fig. 8. This difference can be reduced by approximating the transient temperature by a second order response, which represents the competition between the transport of fluids and the heat transfer through the separating wall. However, the approximation by a first order response with a time constant taking into account globally the transport and the heat transfer seems to be more attractive and present the advantage to assess directly the global transient behaviour. In addition, the advanced control using a simple first order model based on a set of equations with physical significance presents the convenience to be implemented for on-line adaptation particularly when the model parameters depend on the variable of control. This is the case of industrial applications of heat exchangers monitored by the flow rate. Note that the model errors are generally compensated by the use of models in a feed back loop that desensitises their accuracy provided they are based on physical representation but not on black box.

These results lead to deduce the condition, which allows to distinguish between the first and the second case. Response time at cold fluid outlet is always larger or equal to that obtained at its inlet. This means that the expression (18) is only valid when $\tau^*_{\mathrm{c,in}} \leqslant \tau^*_{\mathrm{c,out}}$. This condition allows to choose the appropriate analytical expression to evaluate the time constant of the fluid not submitted to a flow rate step.

Fig. 9 represents the two fluid temperature time constants at the heat exchanger outlets $\tau^*_{\mathrm{h,out}}$ and $\tau^*_{\mathrm{c,out}}$ for positive and negative values of ΔV^*. Theoretical results corroborate the experimental data for the cold fluid. Nevertheless the results for the hot fluid are in agreement in the first case and present some differences in the second case. Note that, for each fluid, time constant obtained for positive or negative step of flow rate, with similar amplitude, presents an asymmetrical variation. As shown in Fig. 9, time constant of cold fluid increases with step magnitude in the second case while it decreases in the first case. Therefore, τ^*_{c} presents a maximum value. The time constant of the hot fluid decreases with step magnitude in all cases.

In order to compare our results to those given in the literature, Table 2 represents three examples of time constants given in references [4] and [8]. These references indicate a unique time constant for both fluids. In the first and second examples presented in Table 2, the time constant values given in the cited references are near the values calculated from the analytical expression corresponding to the fluid submitted to a flow rate step. But in the third example, the time constant of reference [4] is near the value of the fluid not submitted to step of flow rate.

5. Conclusion

Transient response along a counter current heat exchanger is investigated when mass flow rate is subjected to sudden change. The dynamic temperature is approximated by a first order response with a time constant. The hot fluid subjected to the flow rate step presents a decreasing time constant as a function in the flow direction. The cold fluid, which is not submitted to the flow rate step, presents two types of response. The first one corresponds to an increasing response time along the longitudinal axis according to counter flow. In the second case, the cold fluid presents the same dynamic along the heat exchanger. Taking into account the boundary conditions on transient behaviour, analytical expressions of spatial linear variation of response time are deduced for the two cases. Theoretical results are validated by experimental data. The dynamic along the heat exchanger could be characterised by the knowledge of initial and final steady-states temperature profiles. The two cases can be distinguished by the condition, which allows to choose of the appropriate analytical expression of time constant. The influence of flow rate step magnitude on transient behaviour of the heat exchanger is presented. The comparison between time constant to positive and negative flow rate step shows asymmetrical response of the heat exchanger.

Table 2
Comparison of analytical time constants with results given in the literature

	Inner fluid		Outer fluid		Inner fluid		Outer fluid	
	$\dot{m}^0$ (kg s^{-1})	$\dot{m}^\infty$ (kg s^{-1})	$\dot{m}^0$ (kg s^{-1})	$\dot{m}^\infty$ (kg s^{-1})	τ_{out} (s, authors)	τ_{out} (s, authors)	τ_{exp}	τ_{th}
1	0.050	0.180	0.320	0.320	7.0	11.7	5.3 Ref. [8]	5.3 Ref. [8]
2	0.150	0.067	0.100	0.100	26.3	44.6	27.0 Ref. [4]	28 Ref. [4]
3	0.037	0.044	0.097	0.097	32.8	45.2	42.0 Ref. [4]	45.0 Ref. [4]

References

[1] P. Pierson, J. Padet, Etude théorique et expérimentale des échangeurs en régime thermique instationnaire. Simulation d'une phase de relaxation, Int. J. Heat Mass Transfer 31 (8) (1988) 1577–1586.

[2] P. Pierson, J. Padet, Evaluation des transferts thermoconvectifs en régime instationnaire. Approche théorique et expérimentale, Rev. Gén. Therm. Fr. 287 (1985) 781–788.

[3] M. Lachi, N. El Wakil, J. Padet, Comportement transitoire des échangeurs bitubes fonctionnant avec débits variables, Colloque Interuniversitaire Franco-Québecois, Thermique des Systèmes à Température Modérée, Sherbrooke, Canada, 1995, pp. 139–144.

[4] D. Azilinon, P. Pierson, J. Padet, Constante de temps des échangeurs thermiques, Rev. Gén. Therm. Fr. 338 (1990) 64–78.

[5] M. Henrion, M. Feidt, Comportement en régime transitoire de divers types d'échangeurs de chaleur; modélisation et conséquences, Int. Commun. Heat Mass Transfer 18 (1991) 731–740.

[6] M. Hadidi, M. Guellal, J. Padet, Modèle de réponse à deux paramètres pour les échangeurs soumis à des températures d'entrée variables, Colloque Interuniversitaire Franco-Québecois, Thermique des Systèmes à Température Modérée, Toulouse, France, 1993, pp. 139–144.

[7] M. Hadidi, M. Guellal, M. Lachi, J. Padet, Loi de réponse d'un échangeur thermique soumis à des échelons de températures aux entrées, Int. Commun. Heat Mass Transfer 22 (1) (1995) 145–154.

[8] M. Lachi, N. El Wakil, J. Padet, The time constant of double pipe and one pass shell-and-tube heat exchangers in the case of varying flow rates, Int. J. Heat Mass Transfer 40 (9) (1997) 2067–2079.

[9] Y.A. Gögüs, Ö.E. Ataer, Transient of heat exchangers, in: J. Padet, F. Arinç (Eds.), in: Transient Convective Heat Transfer, Begell, New York, 1997, pp. 289–303.

[10] R.K. Shah, The transient response of heat exchangers, in: S. Kakaç, A.E. Bergles, F. Mayinger (Eds.), in: Heat Exchanger Thermal-Hydraulic Fundamentals and Design, Hemisphere, Washington, DC, 1981, pp. 915–953.

[11] F. Fragnaud, Unsteady heat transfer modeling of heat exchangers, in: J. Padet, F. Arinç (Eds.), in: Transient Convective Heat Transfer, Begell, New York, 1997, pp. 329–340.

[12] S.V. Patankar, D.B. Spalding, A calculation procedure for the transient behaviour of shell and tube heat exchangers, in: Afgan, Schlinder (Eds.), in: Heat Exchangers, McGraw-Hill, New York, 1974, pp. 155–175.

[13] J.F. Sacadura, Initiation aux Transferts Thermiques, edn techniques et documentation, 1982.

[14] S. Kakaç, H. Liu, in: Heat Exchangers: Selection, Rating, and Thermal Design, CRC Press, Boca Raton, 1998.

[15] E.N. Sieder, G.E. Tate, Heat transfer and pressure drop of liquids in tubes, Ind. Eng. Chem. 28 (12) (1936) 1429–1435.

[16] H. Kawamura, Analysis of transient turbulent heat transfer in an annulus: Part I: heating element with a finite nonzero heat capacity and no thermal resistance, Trans. JSME 39 (324) (1973) 2498–2507.

[17] C.C. Lakshmanan, O.E. Potter, Dynamic simulation of a countercurrent heat exchanger modelling-start-up and frequency response, Int. Commun. Heat Mass Transfer 21 (3) (1994) 421–434.

[18] R.W. MacCormack, Numerical solution of the interaction of a shock wave with a laminar boundary layer, in: M. Holt (Ed.), Lecture Notes in Physics, vol. 8, Springer, New York, 1971.